WAVE VELOCITIES AT DEPTHS BETWEEN 50 AND 600 KILOMETERS†

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ABSTRACT

Wave velocities in the interior of the earth are usually calculated by means of the Wiechert-Herglotz method. Incorrect interpretation of the form of the travel time curve for a certain distance interval then leads to errors not only in results concerning the corresponding depth interval but (in decreasing amounts) also for all greater depths. A new method is described and applied to that portion of the earth's mantle in which earthquake foci exist. Values obtained with this method for velocities at various depths are independent of each other.

Method Used

In 1906 Wiechert (1907, eq. 291) developed a simple relationship between the velocity V at the deepest point reached by an elastic wave through the earth's interior and the apparent velocity $\bar{V} = d\Delta/dt$ at the epicentral distance Δ (in km.) at which this wave arrives at the surface of the earth. S. Mohorovičić (1914, eq. 21; see also Gutenberg, 1932, eq. 209) applied this equation to the ray which leaves the hypocenter of the shock at the depth h horizontally and arrives at the distance Δ^* (corresponding to θ^* in degrees) at which the travel-time curve has its point of inflection, and where the apparent velocity has its minimum value V^*. Thus he found that

$$V = V^* r/R \tag{1}$$

where $r = R - h$ and R = radius of the earth. Apparently it has not been realized that this equation can be used to find V as a function of h, unless the focus is in a layer in which the velocity decreases with depth at a rate in excess of the critical value given by $dV/dr = V/r$. If the velocity decreases by a greater rate, the wave leaving the source horizontally remains in the low-velocity layer, and the travel-time curve consists of two branches with opposite curvature and without point of inflection. Obviously, in addition, the depth of focus of any earthquake used must be well known, and a sufficient number of reliable arrival times of P or S must be reported from distances between a few degrees less than θ^* to a few degrees beyond θ^*. Neither the distance θ^* nor the origin time of the earthquake is needed for use of the method.

Figure 1 contains several graphs which aid in the application of the method. Curve (a) gives the approximate distance θ^* of the point of inflection in degrees as a function of h; it is based on earlier results. Curve (b) permits finding of the focal depth h in km. if the focal depth below 33 km. is given in fractions of the radius of the earth as used in the International Seismological Summary (see, e.g., International Seismological Summary for 1937, p. 3).

To facilitate use of a large scale in plotting the travel times t as a function of the epicentral distance θ the quantity $p\theta$ is subtracted from all observed travel times; p is selected in such a way that the value $t - p\theta$ is approximately constant in the neighborhood of θ^*. Suitable values of p for the P and S waves as a function of the focal depth h are given in curves (d) and (e) in figure 1.

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Shocks with useful data were selected from the International Seismological Summary. Observations with residuals of about 8 seconds or more were rejected. Times given for two or more stations at the same distance as well as data from two or three stations differing only by a small fraction of a degree were averaged, with consideration for the effect of distance. The factor p was taken from curves (d), for P, and (e), for S, in figure 1, and the quantity $p\theta$ was calculated for all observations in the required distance interval. The differences $t - p\theta$ were plotted separately for P and S as function of θ. Figure 2 gives examples of results. In each instance

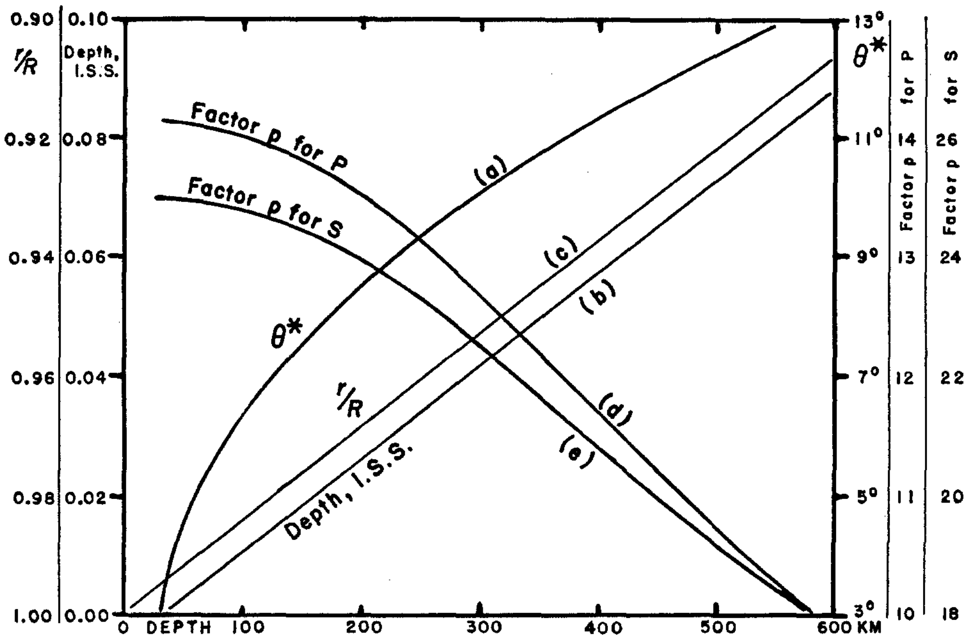

Fig. 1. Curve (a), Approximate epicentral distance θ^* of the point of inflection; (b), focal depths given in the International Seismological Summary; (c) ratio r/R (see text); (d) and (e), factors p for P and S waves respectively. All quantities are given in terms of the focal depth h in km.

two or three arrows indicating specific velocities were drawn to facilitate the finding of the apparent velocity V^* near the point of inflection of the curve fitting best the plotted values $t - p\theta$ (see fig. 2).

For the focal depths h the values given by Gutenberg and Richter (1949) were usually taken. However, when these values differed by more than 30 km. from those given in the International Seismological Summary, h was redetermined, and the revised values were taken.

Materials Used

The Japanese sector is the only region where very deep earthquakes occur for which the data are sufficient for employment of the method described above; it also contains the great majority of useful shocks at intermediate depth. In addition, intermediate shocks in the eastern and central Mediterranean, in Rumania, and in the Hindu Kush were investigated, but for only a few of these were the reported data sufficient to give significant results with the new method.

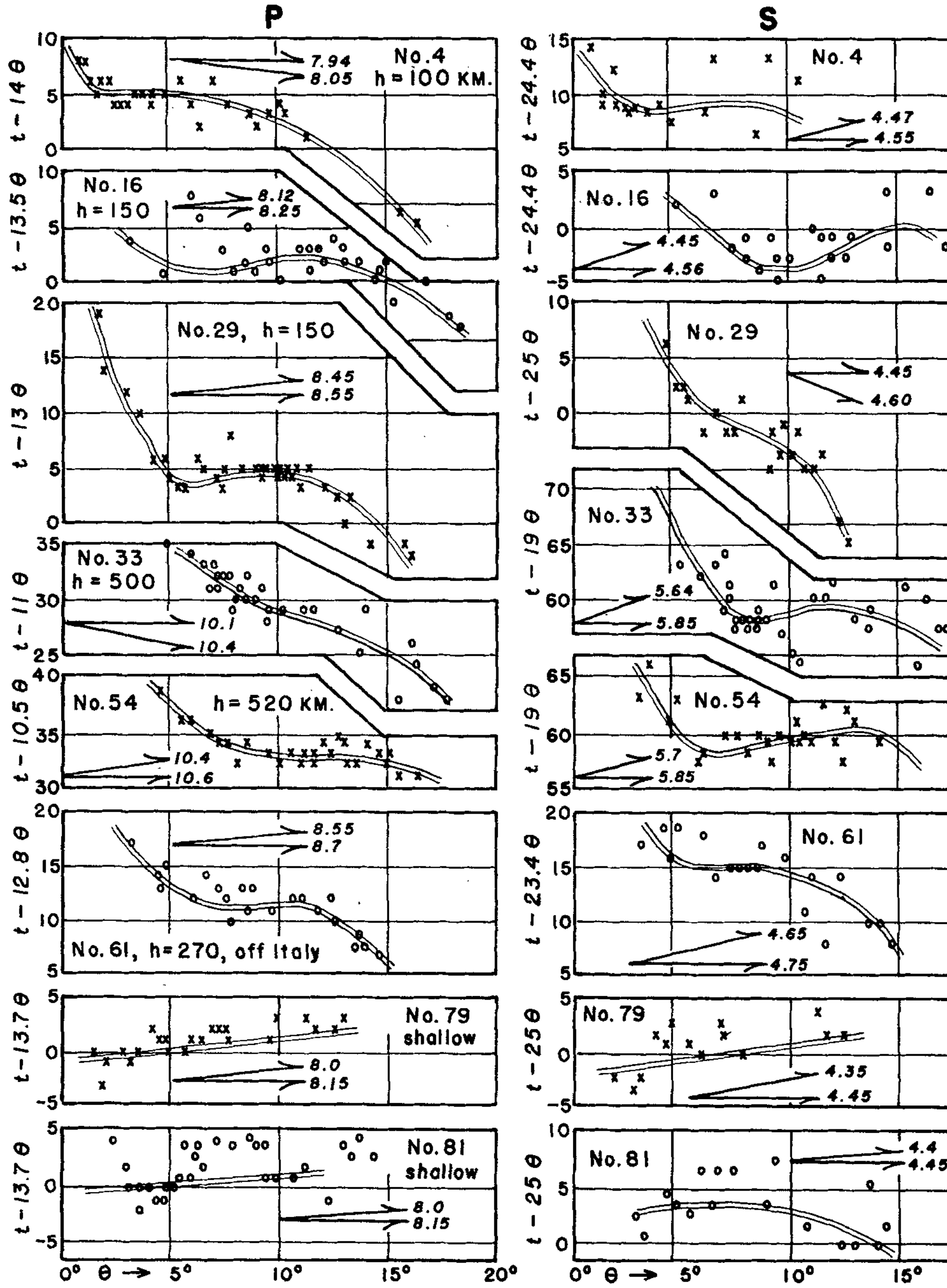

Fig. 2. Examples for determination of the apparent velocity V^* (in km/sec.) at the point of inflection of travel-time curves: left, for P; right, for S. Note that the ordinates are $t - p\theta$, where p is different in each case (see text).

TABLE 1

FUNDAMENTAL DATA ON SHOCKS USED FOR QUALITIES A, B, C

No.	Date	Time	Lat. N	Long. E	h	M	Velocity V in km/sec. P	Velocity V in km/sec. S	Poisson's ratio σ
		min. sec.	deg.	deg.	km.				
1	1931 June 2	2:37	36	137	260	6½	8.2 B	C	C
2	1937 July 26	19:56	38	141	90	7.1	7.85 A	4.4 B	0.27 B
3	1928 Mar. 29	5:06	32	138	410	7.1	9.25 B	5.1 B	0.28 B
4	1938 Feb. 7	14:43	36	139	100	6½	7.8 A	4.4 A	0.26 A
5	1934 April 6	19:09	37	142	100	6¾	7.9 A	C	C
6	1933 July 20	23:14	39	145	100	6¾	7.8 A	4.35 B	0.27 B
7	1931 Mar. 29	17:52	43	144	100	6¾	8.0 A	4.5 A	0.27 A
8	1930 Dec. 23	23:55	43	143	150	6	8.1 B	C	C
9	1931 Jan. 21	8:58	43	146	120	6¼	7.9 A	4.5 A	0.26 A
10	1935 Oct. 2	5:33	44	146	70	7.0	7.8 B	4.5 B	0.25 B
11	1939 Dec. 16	10:46	44	148	75	7.1	7.8 A	4.5 A	0.25 A
12	1934 June 13	11:51	44	148	90	6.9	7.8 A	C	C
13	1940 July 4	9:00	44	144	250	5¾	8.4 A	C	C
14	1938 Oct. 17	15:27	44	140	250	6½	8.2 A	4.45 A	0.29 A
15	1930 July 22	19:26	45	147	130	7.1	7.9 A	4.45 A	0.265 A
16	1936 Nov. 12	20:05	46	148	150	6½	7.9 A	4.35 B	0.28 B
17	1937 June 8	18:01	47	150	180±	6½	8.2 B	C	C
18	1951 July 11	18:22	28	140	480	7	9.1 B	C	C
20	1931 June 29	16:43	34	137	380	6½	9.0 A	5.0 B	0.28 B
21	1936 Oct. 26	9:33	35	136	380	6¼	8.8 A	5.0 A	0.27 A
22	1932 May 5	4:11	35	135	380	6½	8.6 B	C	C
23	1929 June 2	21:38	35	137	360	7.1	8.7 B	C	C
24	1932 July 25	8:25	35	136	360	6¾	8.7 B	4.9 B	0.27 B
25	1936 Oct. 19	19:56	37	135	350	5¾	8.7 B	C	C
26	1935 Oct. 15	14:35	38	135	330	5¾	8.8 B	4.8 B	0.28 B
28	1935 May 31	8:19	39	134	450	6½	9.6 A	5.25 A	0.29 A
29	1936 Dec. 1	6:09	30	129	270	6½	8.1 A	4.4 A	0.29 A
32	1932 Oct. 1	15:06	28	140	450	6	9.4 A	5.0 B	0.31 B
33	1937 Jan. 5	11:09	28	139	500	6	9.4 A	5.3 A	0.28 A
34	1932 Feb. 3	7:34	29	140	400	6½	9.3 B	C	C
35	1933 Sept. 2	16:41	30	139	410	6¾	9.5 B	5.2 B	0.28 B
36	1934 April 19	16:13	30	139	430	6½	9.6 A	5.25 A	0.29 A
37	1932 April 4	19:16	30	139	430	6¾	9.8 A	5.4 A	0.28 A
38	1940 Nov. 7	13:58	30	138	500	6¾	9.1 B	5.1 B	0.27 B
39	1933 Feb. 9	3:57	32	138	370	6	9.1 B	4.8 B	0.30 B
40	1936 June 25	16:52	32	138	370	6¼	9.2 A	4.9 A	0.30 A

TABLE 1—*Continued*

No.	Date	Time	Lat. N	Long. E	h	M	Velocity V in km/sec. P	Velocity V in km/sec. S	Poisson's ratio σ
		min. sec.	deg.	deg.	km.				
41	1933 Mar. 18	15:52	32	140	170	6½	7.7 A	4.35 A	0.27 A
42	1937 May 28	19:56	24	142	530	6½	9.7 A	5.3 B	0.29 B
43	1933 Mar. 11	19:33	26	141	510	6¾	9.7 A	5.2 B	0.31 B
44	1940 Mar. 9	10:47	27	140	500	6½	9.6 A	5.2 B	0.30 B
45	1933 Sept. 9	5:03	44	130	590	6¼	10.1 A	5.6 A	0.28 A
46	1935 Mar. 28	23:48	43	131	550	6¼	10.0 B	5.6 B	0.28 B
47	1940 July 10	5:50	44	131	560	7.3	9.8 A	5.3 A	0.29 A
48	1931 Feb. 20	5:33	44	136	350	7.4	9.0 A	4.8 A	0.30 A
49	1932 Nov. 13	4:47	44	137	320	7.0	8.6 A	4.65 A	0.29 A
50	1937 April 29	20:19	46	137	370	6¼	8.6 A	4.8 A	0.28 A
51	1932 Sept. 23	14:22	45	138	300	6.9	8.85 A	4.75 A	0.30 A
52	1932 Oct. 25	17:02	47	144	400	6½	8.8 A	4.85 A	0.29 A
53	1933 Dec. 4	19:34	47	144	360	6¾	8.6 A	4.75 A	0.28 A
54	1939 April 21	4:29	48	139	520	7.0	9.7 A	5.25 A	0.30 A
55	1933 May 24	4:36	48	146	420	6	8.9 A	4.85 B	0.29 B
56	1935 July 26	8:04	48	145	480	6½	9.35 A	5.05 A	0.29 A
57	1936 Mar. 1	10:22	48	147	430	6¼	9.0 A	4.85 B	0.31 B
58	1935 Mar. 18	8:41	36	27	130	6¼	8.0 A	C	C
59	1934 Nov. 9	13:41	37	26	140	6¼	7.7 B	C	C
60	1935 Feb. 25	2:52	36	25	80	6¾	8.0 B	C	C
61	1938 April 13	2:46	39	15	270	6¾	8.2 A	4.55 A	0.28 A
62	1934 Mar. 29	20:07	46	27	150	6¼	7.8 A	4.35 B	0.28 B
63	1938 July 13	20:15	46	27	150	5¼	8.0 B	C	C
64	1939 Sept. 15	6:02	46	27	150	5¼	8.0 B	C	C
65	1940 Oct. 22	6:37	46	27	150	6½	8.0 A	4.4 B	0.28 B
66	1940 Nov. 10	1:39	46	27	140	7.4	8.2 A	4.45 B	0.29 B
67	1936 June 29	14:30	36	71	230	6¾	8.4 B	4.55 B	0.29 B
68	1937 Nov. 14	10:58	36	71	240	7.2	8.25 B	C	C
69	1939 Nov. 21	11:02	36	71	220	6.9	8.3 B	C	C
70	1940 May 27	4:11	37	71	240	6¼	8.2 B	C	C
71	1931 June 2	2:38	36	137	260	6½	8.2 B	C	C
72	1934 April 13	22:04	26	125	250	6	8.1 B	C	C
73	1932 Dec. 26	21:15	26	125	210	6¼	8.3 A	C	C
74	1930 Dec. 21	14:51	20	122	170	6.9	8.2 A	C	C
75	1938 Sept. 21	18:52	36	141	60	6.9	7.8 A	4.45 B	0.25 B
76	1938 May 23	7:18	36	141	(40)	7.4	7.8 A	4.4 B	0.26 B
77	1938 Nov. 6[a]	8:54	37	142	60	7.6	8.1 A	4.45 A	0.27 A
79	1933 June 18	21:37	38	143	(40)	7.3	8.0 A	4.4 A	0.27 A
80	1939 Oct. 10	18:32	39	143	(40)	7.4	7.7 A	4.4 A	0.26 A
81	1933 Mar. 2	17:31	39	145	(40)	8.5	8.0 A	4.45 A	0.27 A
82	1937 Feb. 21	7:03	45	149	(40)	7.4	8.2 A	C	C

[a] Only data from 1½ to 6 degrees distance used.

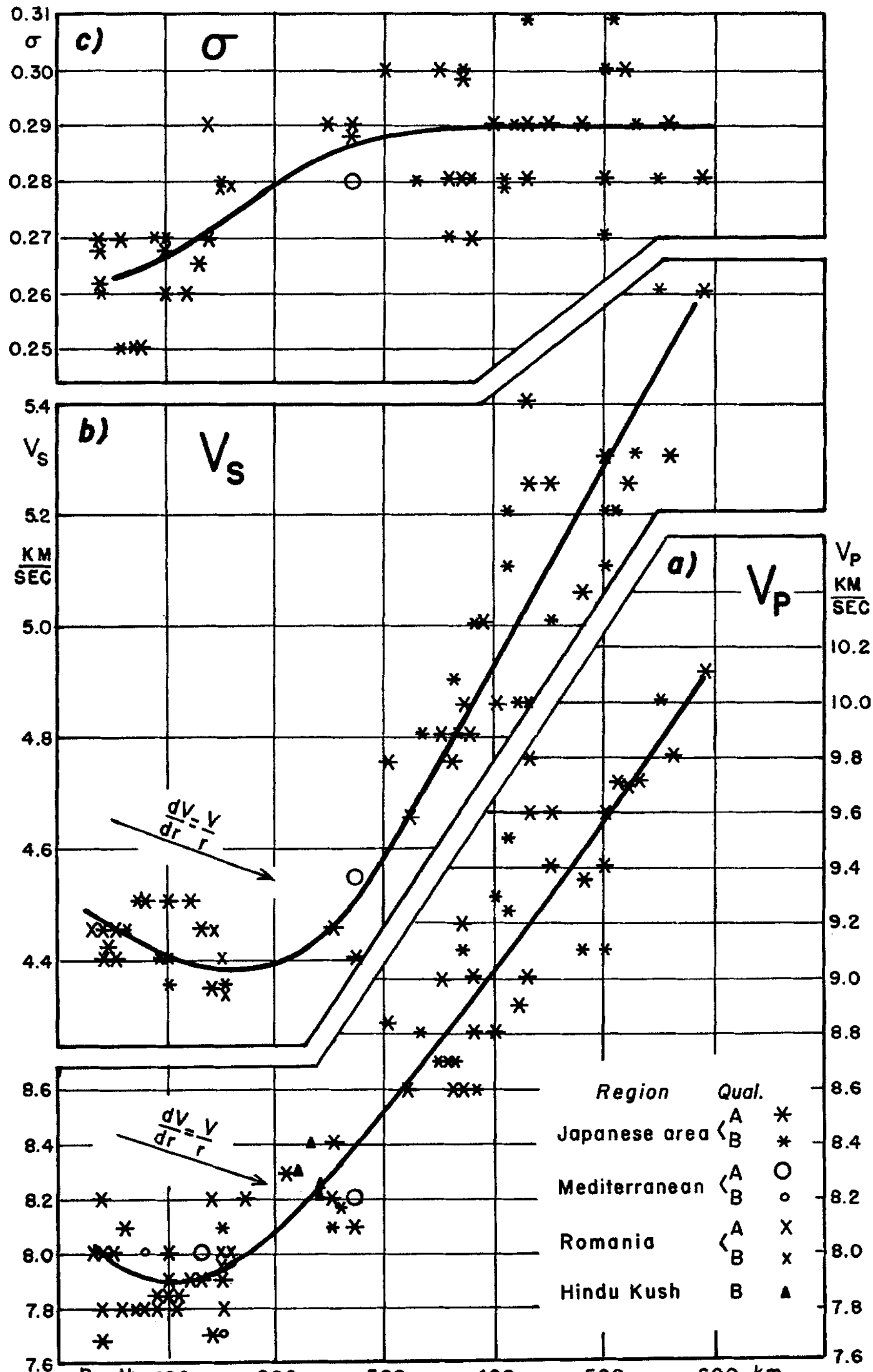

Fig. 3. Velocities of P and S waves and Poisson's ratio as a function of the depth h. Data are taken from table 1.

Most shocks used in the present research occurred between 1930 and 1940. Prior to 1930 there were not enough stations in Japan with accurate time service, and beginning with 1940 at many stations accuracy of the reported times deteriorated as a consequence of the war. Furthermore, for later years the International Seismological Summary is not yet available, so that data have to be collected from station bulletins and all epicentral distances must be calculated (e.g.,in no. 18 of table 1).

Fundamental data of all shocks which furnished significant results are summarized in table 1. For identification of the shocks their origin times (to the nearest minute) and epicentral coördinates (to the nearest degree) are given. Velocities at the depths h of the foci are calculated from equation (1). Qualities Q are added as follows: A, when V is probably accurate within ± 0.1 km/sec.; B, when V seemed

TABLE 2

AVERAGE TRAVEL TIMES t IN MIN:SEC. OF Pn IN JAPAN AT THE DISTANCES θ IN DEGREES (After Wadati and Oki [1933, p. 124] and calculated values $t - 14\theta$ in seconds)

θ	3	4	5	6	7	8	9	10	11	12
t	0:52	1:06	1:21	1:34	1:49	2:03	2:17	2:30	2:44	2:58
$t - 14\theta$	10	10	11	10	11	11	11	10	10	10

to be accurate within ± 0.2 km/sec.; C, when the data seemed to be insufficient for a determination of V within ± 0.2 km/sec. Results of quality C were omitted from table 1 and figure 3. Poisson's ratio σ at the depth h was determined for each shock from the ratio of the apparent velocities V^* of P and v^* of S. The quality Q of σ corresponds to the lower in the two preceding columns.

The results given in table 1 should be supplemented by the wave velocities just below the Mohorovičić discontinuity. Determinations of these values from blasts and shallow earthquakes show rather large discrepancies as a consequence of the small amplitudes of Pn and especially Sn at the epicentral distances involved. While earlier data have resulted in velocities of about 7.8 and 4.4 km/sec., respectively, recent records written by more sensitive instruments give everywhere velocities near 8.1 or 8.2 km/sec. for Pn and near 4.5 km/sec. for Sn. In Japan where no highly sensitive instruments are in use, the velocities found for Pn from records of shallow near-by earthquakes are between about 7.7 and 8.0 km/sec. From travel times reported for several large shocks Wadati and Oki (1933) worked out average travel times from which the data in table 2 were taken. This table also gives $t - 14\theta$ (θ = epicentral distance). Since this is constant with the limits of error, a velocity near 111.11/14 or 7.94 km/sec. is indicated for Pn.

For a few large shocks between 1931 and 1939 the velocities of Pn and Sn were investigated by plotting $t - p\theta$ and applying a procedure similar to that described for finding of V^* in the deep shocks. The resulting velocities are entered in figure 3. Data for two of these shocks are reproduced in figure 2 (Nos. 79 and 81). Considering all facts, velocities of 8.0 for Pn and 4.45 km/sec. for Sn were indicated in figure 3 as corresponding best to the velocities found for depths greater than 40 km. The actual velocities at depths between 50 and 250 km. may turn out to be slightly higher, when more records of highly sensitive instruments become available and permit the finding of the first small waves of P and S in records of shallow and intermediate shocks at distances between about 5° and 20°.

A decrease in velocity with depth at a rate greater than that given by $dV/dr = V/r$ will produce a shadow zone at the surface (for details see Gutenberg and Richter, 1939). If $V = 8.0$ km/sec. for P and $v = 4.45$ km/sec. for S, the critical rates of decrease are respectively 0.13 km/sec. and 0.07 km/sec. per 100 km. The critical slopes of the velocity-depth curves are indicated by arrows in figure 3. It is evident that the actual rate of decrease in velocity may well exceed the critical value at

TABLE 3

VELOCITIES V OF P AND v OF S AND POISSON'S RATIO σ AS A FUNCTION OF DEPTH h
(N = new values from figure 3—mainly Japan; G = Gutenberg, 1948; J = interpolated from Jeffreys, 1939, pp. 511, 512)

h (km.)	V km/sec.			v km/sec.			σ N
	N	G	J	N	G	J	
50	8.0	8.0	7.8	4.45	4.46	4.36	0.26
100	7.85	7.9	7.95	4.4	4.44	4.45	0.27
150	7.9	7.9	8.1	4.35	4.46	4.5	0.27
200	8.1	8.1	8.3	4.4	4.5	4.6	0.28
250	8.3	8.4	8.45	4.45	4.6	4.7	0.29
300	8.5	8.6	8.6	4.6	4.7	4.75	0.29
400	9.0	9.1	9.0	4.95	4.95	4.9	0.29
500	9.6	9.6	9.7	5.3	5.3	5.3	0.29
600	10.2	10.1	10.3	5.65	5.6	5.65	0.29

certain depth intervals between 50 and 200 km., especially for S. In this case, the velocity resulting from equation (1) may correspond to a diffracted wave over a different path and may be higher than the actual velocity at the corresponding depths. If the velocity decreases by a rate close to the critical value, the amplitudes and ray paths are very sensitive to small changes in velocity. In this case, relatively great differences in amplitudes at a given distance can be expected for P and S waves in different regions for earthquake records written at epicentral distances between 5° and 20° and for shocks with focal depths of less than about 200 km. In one region the travel-time curve may be continuous, in another it may consist of two separate branches. Especially S waves traveling in near-horizontal directions may penetrate in some regions appreciably deeper than in others before they turn upward again. Such regional differences are known to occur (see, e.g., Gutenberg, 1948, pp. 143, 145, and Lehmann, 1952).

Average values for the velocities and Poisson's ratio σ in the upper 600 km. of the earth's mantle which are indicated by the curves in figure 3 are compared with older results in table 3. They agree within the limits of error so far as their absolute values are concerned in spite of the fact that the older data are based on travel times

t in shallow shocks as a function of the epicentral distance θ from $\theta = 0$ to a given distance, while the new values depend on the minimum value of $d\theta/dt$ as a function of the focal depth of various earthquakes.

The new curves for the velocities as a function of depth bring out prominently the "low-velocity layer" which has been suggested previously, mainly on the basis of the changes of amplitudes with distance (for a historical review see, e.g., Gutenberg, 1948). There is no clear indication of a discontinuity in figure 3.

Poisson's ratio σ appears to increase gradually with depth in the upper 300 km. of the mantle, as has been found previously (see, e.g., Wadati, 1933; Gutenberg, 1948). This, as well as a comparison of the curves for P and S in figure 3, shows that in the outer part of the mantle the rigidity increases more slowly with depth (or decreases faster) than the bulk modulus. This is most likely a consequence of the increase in temperature modified by the increase in pressure. Whether a change in phase or composition is involved, or both, or neither, cannot be decided from the results.

Summary

A new method is described for finding the velocities of longitudinal and transverse waves in the upper 600 km. of the earth's mantle. It is based on the apparent velocity ($d\Delta/dt$) at the point of inflection of the travel-time curve as a function of the focal depth of earthquakes. The resulting velocities below the Mohorovičić discontinuity show a clear decrease with depth with a minimum at a depth of roughly 100 km. for longitudinal waves and 150 km. for transverse waves. Poisson's ratio increases from about 0.26 at a depth of 50 km. to 0.29 at 250 km. and does not change noticeably in the deeper part of the earth's mantle. Small local differences in the rate of the velocity decrease result in noticeable local differences in amplitudes and perhaps even in travel times at epicentral distances between about 5° and 20°. There is no evidence of a discontinuity in the mantle between the low-velocity layer and a depth of about 900 km., nor of noticeable differences in velocity at any given depth between the various regions for which data are available.

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